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Faculty Working Papers

THE MARKET MODEL: POTENTIAL FOR ERROR

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College of Commerce and Business Administration
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IMPLEMENTATION OF MECHANISMS BY PROCESSES:
PUBLIC GOOD ALLOCATION EXPERIMENTS

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Summary:

    The design problem for decentralized public good provision is outlined,
hypothesized to be incomplete as specified in the literature. Experimental
evidence is presented which supports the sensitivity of allocations to the
process, or informational logic, which is employed in public good decision-
reaching.

A traditional view in economics has held the free-rider problem to be an insurmountable barrier to the Pareto-efficient provision of public goods via voluntary, decentralized decisionmaking. When announced voluntary cost shares are collected as a means of obtaining resources for public good production, individuals face a conflict between correctly revealing marginal evaluations and obtaining a low share of public production cost.

An alternative, less pessimistic viewpoint arises in the recent literature on allocation mechanisms.[1] Viewing decentralized decisionmaking as a design problem, various authors have constructed mechanisms which can achieve Pareto-efficient allocations as Nash equilibria.

This paper presents experimental evidence suggesting that the design problem is incompletely specified in the theoretical literature. Section 1 outlines the design problem and defines the major element which is unspecified in theoretical analyses, the process, an informational logic by which an allocative decision is reached. Section 2 reviews Vernon Smith's experiments and introduces the Seriatim process, an alternative to the process used by Smith. Experimental procedures are related in Section 3, and results reported in Section 4. A discussion section concludes the paper.

I. The Design Problem

Let $\theta = \{1,2,\ldots,i,\ldots,I\}$ be the set of economic agents who will consume some allocation $y \in Y$, a set of feasible allocations. While more generality is possible,[2] it is convenient to presume $y = (y_0, y_1, \ldots, y_I)$, where y_0 is the quantity of a pure public good, and y_i a private allocation to agent i. Each agent i has a utility function $u_i \in U$, a space

of utility functions. We denote $u = (u_1, \ldots, u_I) \in U^I$, where u is called a utility profile, and the space of utility profiles is the I-fold product of U. An economic environment (u,Y) is described when preferences and possibilities are specified.

A performance function $g(\cdot) : U^I \to Y$ selects allocations satisfying performance criteria given preferences. The usual performance function is the Paretian, which, given any utility profile, picks out Pareto-efficient allocations from Y. If preferences are known and the designer controls allocations, he satisfies these performance criteria by directly allocating in accordance with $g(u)$.

It is extreme, however, to model the designer as capable of directly observing u. Typically, the designer observes only messages sent by agents, modelled as choice of $m_i \in M$, a space of feasible messages. Again, let $m = (m_1, \ldots, m_I) \in M^I$ be a message profile. Now the designer specifies an outcome function $f(\cdot) : M^I \to Y$, where the allocation resulting is dependent upon observed messages, not unobservable preferences. The designer's choice then, is of a feasible message space, M, and an outcome function, f, together called an allocation mechanism, $D = (M,f)$.

The messages sent by agents depend upon the mechanism D being employed, and upon what messages other agents send. This dependence is summarized by a joint behavior rule, $e(\cdot, D) : U^I \to M^I$, predicting a message profile given a utility profile. The standard $e(\cdot, D)$ is Nash equilibrium behavior.

The design problem: to find an allocation mechanism D^* such that for a given joint behavior rule $e(\cdot, D^*)$, the performance function is

decentralized by the mechanism. That is, find a D* satisfying
$g(u) = f*[e(u,D*)]$ for any $u \in U^I$.

Groves and Ledyard [1977] have the seminal positive result on this
problem, constructing a solution when the joint behavior is Nash equil-
ibrium.

However, specifying the mechanism may not be sufficient to predict
joint behavior given the utility profile. In particular, it is con-
ceivable that joint behavior may also depend upon what we call the
process.

Definition: A process is a formal informational logic of decision
reaching which specifies: 1) a starting rule, including a determination
of the degree of information initially processed by each agent; 2) a
continuance rule which determines the timing of message relay or recon-
sideration, and of additional information transmitted to each player;
and 3) a stopping rule which ordinarily consists of an operational defi-
nition of agreement outcome and institution, and a default outcome to
be instituted if agreement is not reached within some unambiguously
measurable time horizon.

Since to some extent the process employed may affect an agent's per-
ception of the possibility, profitability, or strategic viability of
messages, the process may be hypothesized to affect joint behavior result-
ing from a given mechanism. Experimental data below afford a preliminary
test of this hypothesis.

It seems appropriate to consider the role of a process, if this
hypothesis is supported, to be an attempt to implement mechanisms. For-
mally, a mechanism may be said to be implemented by a process if the

outcomes of joint behavior obtained by the mechanism-cum-process approx-
imately attain the performance standard (are approximately Pareto-
efficient).

II. Smith's Experiments

Vernon Smith[3] pioneered the experimental verification of economic
theory, and is principally responsible for the development of an experi-
mental economic methodology.

His initial public good experiments employed the Groves-Ledyard
mechanism (a partial equilibrium version called "optimal") and what we
choose to call the Smith process. Specifically,

Groves-Ledyard mechanism: Player $i=1,2,\ldots,I$ chooses a message
$n_i \in G \subset R^+$, interpreted as an increment to public good production. De-
noting total public good production by $T = \sum_{i=1}^{I} n_i$, the set of feasible
states is

$$F = \{(n_1,\ldots,n_I) \in G^I | T \in G\}.$$

Let $S_i = T - n_i = \sum_{j \neq i} n_j$. The "distribution of cost" function for i is

$$c^i(n_i,S_i) = [(q/I) - IS_i](n_i+S_i) + \frac{(I-1)}{2}(n_i+S_i)^2 + k_i$$

where q is per unit production cost, k_i a constant. This formula may
not be transparent, but has been designed to Groves-Ledyard to support,
in the Nash equilibrium sense, a Pareto-efficient message profile. The
outcome function, f, has component functions

$$f^i(n_i,S_i) = V^i(T) - c^i(n_i,S_i), \quad i=1,\ldots,I,$$

where $V^i(T)$ is the value in dollars to i of T units of the public good.

Smith's public good experiments to date have all employed the same process, which he has not found it necessary to name. As, to our knowledge, it has no clear antecedents in the relevant literature, we choose to call it the Smith process:

Smith process: 1) Starting rule: in period t, each player independently and privately selects a message $m_i(t)$. 2) Continuance rule: a) all players are informed of m(t), then the process proceeds to period t+1; b) the complete stopping rule is known in advance by all players. 3) Stopping rule: a) the process stops in period t* if $m_i(t^*) = m_i(t^*-1)$, all i,m(t*) satisfies agreement and/or feasibility conditions, and $t^* \leq T$; in which case the outcome is f(m(t*)). b) otherwise the process stops in period T (value of T prespecified), and a prespecified default outcome is instituted.

Smith's original experimentation examined the role of incentive comparability, by contrasting experiments with mechanisms for which there is a Pareto-efficient Nash equilibrium, with experiments using theoretically unsound mechanisms. His Groves-Ledyard mechanism experiments supported the hypothesis: if agreement is reached, the proposed public good quantity will be Pareto-efficient. Experiments with a Lindahl mechanism, a specification in mechanism form of the logic of the Lindahl [1919] approach, supported the hypothesis: if agreement is reached, the allocation reached can be Pareto-dominated by another allocation with a greater quantity of the public good.

Additionally, Smith has constructed partial equilibrium and general equilibrium versions of an Auction mechanism for public goods, which are incentive-compatible. The former experimentally approximates Lindahl equilibrium outcomes, while the latter reaches Pareto-efficient outcomes, but with the "wealthy" overcontributing and the "poor" undercontributing relative to Lindahl contributions.

These results lend strong support to the belief that public good allocations are sensitive to the incentive-compatibility properties of the mechanism being employed. To determine whether allocations are also sensitive to the process employed, in particular, to see if the Groves-Ledyard mechanism may be implementable by some processes but not others, we introduce an alternative called the Seriatim process.

<u>Seriatim process</u>: 1) Starting rule: each player i chooses initial message $m_i(0)$ in period 0.

2) Continuance rule: in period t, only player $j = t \bmod I$ ($I = \#$ players) reconsiders his message, all others automatically repeated. (That is, only player 2, for example, reconsiders his message in period 2, all other players are inactive. If there are 5 players, the process recycles in period 6, with only player 3 active in period 8.) Before the choice in period t, the player is informed of the current messages of all other players, or the aggregate of current messages, if that is all that is relevant.

3) Stopping rule: a) If, in any I consecutive periods, each player chooses to repeat his previously chosen message, agreement is reached, and the associated outcome is instituted. This aspect of the stopping rule is known in advance. b) If agreement is not reached within T

periods, the default outcome is instituted. This aspect of the stopping rule is not announced before it is instituted.

III. Experimental Procedures

We conducted six experiments in Evanston and Urbana using an experimental design very similar to that of Smith [1979], but employing the Seriatim process. An unannounced limit of 80 iterations was imposed. Each experiment used three subjects, given evaluations of the public good as shown in Table 1, and instructions including cost and net value tables (as shown in Appendix I). The public good was produceable in facility sizes of $0, 1, \ldots, 11$ at a unit cost of 36.

Cash payments were used to induce valuations of the abstract public good upon subjects. The theory of induced valuation is developed in Smith [1976]. Most subjects were students, with about a half dozen other occupations represented; no subject had economics training beyond a principles class.

Subjects read the instructions privately (except for the first experiment where the subjects commented that reading the instructions aloud was distracting), and any questions were answered, unless they related to behavioral issues or experimenter perceptions. Subjects were paid $2 cash at the beginning of the experiment, and placed in separate rooms for the duration of the session. At each turn t, the monitor opened the door of the active player's room and informed her/him of the current value of S, and withdrew. When the player had chosen a value for n, the monitor recorded the choice on scrap paper and left, closing the door.

Table 1: Valuation Functions

T	v^1	v^2	v^3
0	0	0	0
1	22	23	31
2	42	42	50
3	61	55	61
4	79	67	69
5	93	78	74
6	105	87	78
7	112	92	81
8	116	95	82
9	118	96	82
10	118	96	82
11	118	96	82

T: Facility size

v^i: Total value (in \$) of T units to player i.
(Each player saw only his own column.)

IV. Experimental Results

The six experiments were conducted in accordance with the procedures outlined above. Results obtained are summarized in Table 2. The first row of Table 2 indicates the outcome which would be the Pareto-efficient Nash equilibrium (unique, in this experimental economy), and the next six rows report the outcomes actually observed.

Column 1 reports the number of iterations in each experiment. The next three columns present the final bids of the three subjects, where a bid (n) is a proposed addition to facility size. The final facility size, T, is the sum of proposed additions, shown in column 5. Columns 6-8 report subject earnings (excluding initial \$2). Efficiency figures shown provide the total consumer surplus achieved by all three subjects at the agreed-upon facility size as a percentage of total consumer surplus achievable at the Nash equilibrium allocation (\$71).[4]

Table 2: Experimental Results

| | # of iter. | Subject | | | Facility size | Subject | | | Efficiency |
		1 n	2 n	3 n	T	1 $	2 $	3 $	
NE		3	1	0	4	18	18	18	
#1	13	4	4	0	8	43	10	79	7%
#2	12	3	1	0	4	18	18	18	100%
#3	23	3	2	0	5	29	17	29	92%
#4	79	0	2	0	none	0	0	0	0%
#5	77	3	3	0	6	42	12	43	76%
#6	28	2	1	3	6	60	48	-11	76%

Experiment #4 failed to reach agreement within the allowed (but un-mentioned) 80 iterations.[5] Experiment #2 was the only experiment at-taining the Pareto-efficient Nash equilibrium allocation. All others ended at an allocation which was not a Nash equilibrium, and which con-stituted overproduction of the public good.

After experiment #1, the subjects expressed some surprise that we actually were going to pay them the dollar amount of their earnings. (It appears to be a well-known practice of psychologists at both Illinois and Northwestern to deceive experimental subjects about the amount of money they can earn.) In the remaining experiments, we at-tempted to counteract this tendency by carefully flashing a large roll of bills when the initial $2 was given to each subject.

Outcomes in experiments #4 and #6 clearly resulted from atypical behavior on the part of the subject 3 in each. Their exhibited behavior may not be replicable with any frequency in a large number of trials.[6]

Subject choices over all iterations are classified in Table 3, and in Table 4 for a truncated set of iterations.[7] The classification scheme is that introduced by Ledyard [1978], to examine the question of whether behavior by subjects in the Groves-Ledyard mechanism, Smith process experiments (Smith, 1979) can be explained as a Cournot reaction to the previous messages of the other agents.

In both Tables 3 and 4, the columns denote experiments #1-#6. Row 1 gives the frequency (number of behavioral choices over the total number of choices or iterations) of Cournot behavior (C), which is the selection of that bid which maximizes earnings given the current bids of all other subjects. Row 2 (R) shows the frequency with which subjects repeated their most recently chosen bid. Row 3 (C & R) is the intersection of C and R, the frequency with which repeat bids that were Cournot occurred. Row 4 indicates the relative occurrence of Partial Cournot (PC) bids, which are bids intermediate between the repeating bid and the Cournot bid. In addition to Ledyard's categorizations, we have compiled frequencies for a strategy which might be called Satisficing (S). Satisficing here means (a) repeating the bid when the earnings associated with this are at least 95% of the earnings associated with the Cournot strategy (given the current bids of others), otherwise, (b) choosing the Cournot bid. The final row (U) shows the frequency of bids unexplained by any of the strategies.

V. Discussion of Results

The substitution of the Seriatim process for the Smith process in these experiments appears to introduce a tendency for public good overproduction, relative to Smith's experiments. To test this, we employ

Table 3:   Subject Behavior Over All Iterations

	#1	#2	#3	#4	#5	#6
C	3/13	9/12	15/23	29/79	17/77	11/28
R	5/13	7/12	11/23	37/79	38/77	5/28
C&R	1/13	5/12	7/23	16/79	9/77	4/28
PC	4/13	0/12	0/23	10/79	10/77	7/28
S	5/13	10/12	20/23	42/79	29/77	16/28
U	2/13	1/12	4/23	19/79	21/77	9/28

Table 4:   Subject Behavior Over A Truncated Set of Iterations

	#1	#2	#3	#4	#5	#6
C	3/7	4/6	13/17	26/73	15/71	8/22
R	2/7	3/6	6/17	33/73	33/71	2/22
C&R	1/7	2/6	6/17	14/73	8/71	2/22
PC	1/7	0/6	0/17	10/73	10/71	5/22
S	3/7	4/6	15/17	39/73	26/71	13/22
U	2/7	1/6	4/17	18/73	21/71	9/22

the statistic Q, the ratio of agreed-upon public good quantity to Pareto-efficient Nash equilibrium public good quantity. Assuming that the experimental observations of the statistic Q in the two studies are drawn from arbitrary probability distributions F_S and F_{HM} which are identical save possibly for location, it is possible to test the null hypothesis of identical location using the Wilcoxon two-sample test.[8] The rank-sum of Q_1-Q_3, Q_5, Q_6 from the results above is 38.5/55, statistically significant at p < .025. Thus, significant overproduction occurs for the Groves-Ledyard mechanism, Seriatim process relative to the same mechanism, Smith process.

This provides tentative support for the sensitivity-to-process hypothesis of section 1 above. Specification of the mechanism alone is insufficient to predict public good allocations in these experimental economies. That the Nash equilibrium is less frequently attained with the Seriatim process is somewhat surprising to us, especially since the prior announcement of a maximum number of periods (as in Smith's experiments) would seem more supportive of non-Nash outcomes.

Ledyard [1978] calculates for Smith's experiments the percentage of "explainable" bids (Cournot, Partial Cournot, or Repeat) which are Cournot. For 5-subject experiments, this ratio is 41%, 47%, 50%, and for 8-subject trials 32.5%, 33%. The corresponding statistics for experiments 1-6 above are 27.3%, 81.8%, 78.9%, 48.3%, 30.4% and 57.9%, respectively. The rank-sum for our experiments is 40/66, p < .268. While Cournot behavior appears somewhat more frequently with the Seriatim process, the difference is not so pronounced as to be statistically significant independently of distributional parametrization.

A similar rank sum test on frequency by subject of Cournot choices rejects the null hypothesis that subject behavior in our experiments 2 and 3 is drawn from the same distribution as is our other four experiments (p < .01). This raises the question of whether frequency of Cournot behavior by subjects is related to Cournot bid frequency of the other subjects in the same experiment.[9]

A non-parametric chi-squared cell frequency test rejects (p < .10) the null hypothesis that individual frequency of Cournot choices is un-correlated with Cournot choice frequency of the other two subjects.[10] Analogous rank-sum and chi-squared tests for frequency of satisficing behavior were even more significant, rejecting the hypothesis that fre-quency of subjects' satisficing bids is independent of others' satis-ficing frequencies.[11]

An important part of the explanation of public good overproduction presumably comes from budget-balance considerations. Groves and Ledyard point out that the version of their mechanism does not necessarily col-lect cost shares equal in sum to production cost. We are not aware of a reference to this in the literature, but the nature of the budgetary surplus for all examples we have constructed is a monotonically (dramati-cally so) decreasing function of the quantity of public good produced.

To induce valuations upon subjects and employ the mechanism, it is necessary to pay subjects amounts summing to the total consumers' sur-plus plus the budget deficit. This required us to pay substantially more in the aggregate for production above the Nash equilibrium level.

Bid profiles which lead to overproduction always allow at least one subject to gain through unilateral reduction, and usually involve one

player making less than at the Nash equilibrium. The budget deficit, however, may be crucial to the observed overproduction. We have no insight as to why the Seriatim process may lead the subjects to perceive this opportunity more effectively than the Smith process.

Experiments with another version of the Groves-Ledyard mechanism (the "Quadratic") are now in progress. They balance the budget, paying out in sum the total consumer's surplus, and are general equilibrium in the sense that income effects alter public good demand. These are more complicated experiments requiring the aid of the PLATO computer system.

Footnotes

[1] This literature began in a diffuse fashion, but Hurwicz [1972], Groves and Ledyard [1977], and Malinvand [1971] are major references. Several recent works are in the "Symposium on Incentive Compatibility" which is Volume 46(2), no. 143 of The Review of Economic Studies, April 1979. See Groves [1979] in that issue, or others, for more extensive references.

[2] This presentation of the design problem closely follows Groves [1977].

[3] Smith [1962] is the original private goods market experiment. Smith [1979], originally a 1975 working paper, is the study reported here. Smith also analyzed public good experiments. For an experimental study of discrete public good experiments and references to antecedents, see Ferejohn, Forsythe, Noll, and Palfrey [1979]. The development of an experimental methodology for verification of allocatio theory is in Smith [1976].

[4] Subject earnings equalled the sum of consumers' surplus and the budget deficit. A $17 budget surplus at the Nash equilibrium is built into the cost share formulae.

[5] A non-repeated choice at iteration 79 ended the experiment.

[6] Subject 3 in experiment #4 appeared to derive utility from an uncontrolled phenomenon. He remarked at the beginning that he knew the university did not allow experimenters to require a subject to pay money in an experiment, and he may have derived satisfaction from vicariously obtaining debts he was not obligated to pay. He chose bids leading to large negative earnings (given bids of others) on 23 of 26 occasions, and repeated only once, on iteration 78.

Subject 3 in experiment #6 clearly did not understand the decision procedure, despite a number of instances when clarification was directly offered. She did not ask any questions. After the experiment we discovered she had more than once entered on her session record an amount of earnings which did not correspond to any of the possible earnings on the session record. She had recorded her final earnings as $20 (rather than the correct -$11).

Throughout the paper, our references to particular subjects assume a random gender for pronouns.

[7] Table 4 drops the first 3 iterations to reduce the impact of early learning about the procedure, and the last 3 iterations, reflecting the stopping rule requirement of repeated choices.

[8] Hajek and Sidak [1967]. With the small sample sizes enforced in experimental studies by cost consideration, it is inappropriate to base statistical analysis upon any particular distributional assumptions. Throughout, we restrict ourselves to nonparametric methods.

[9]Noncooperative game theory could predict this. If a strategy
is a mapping from the other players' current bids to your current bid,
your best strategy given the others' strategies will not in general
be the Cournot bid function unless their strategies are their Cournot
bid functions. It is not clear, however, how experimental subjects
could have discerned the strategies of the others.

[10]Own Cournot frequencies and others' Cournot frequencies were
divided into above and below 40%. Chi-squared statistic was 3.6.

[11]Cell division was by above and below 60% satisficing bids.
Chi-squared statistic was 5.5 ($p < .025$). The 40% and 60% dividing
lines have no particular import, and were selected to make expected
cell frequencies as reasonable as possible. The test statistic may
not be distributed approximately chi-squared if expected cell fre-
quencies are too low (Cochran [1954]).

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first part of the paper identifies two schools of